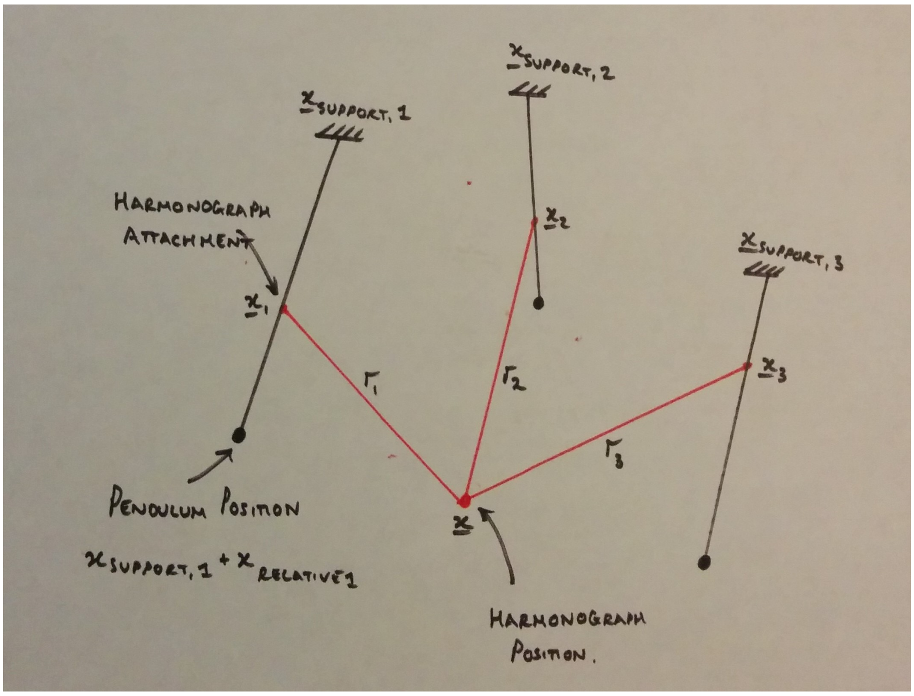
# A simple harmonograph

**Hanging a central 'bob' from three strings, each of which is attached to a spherical pendulum, as below:**




## Finding the position of the three points of attachment between the pendulums and the harmonograph:

**For each pendulum, there is support point $x_{support,i}$, and a position of the pendulum relative to that support point $x_{relative,i}$. The bob for the harmonograph is attached at some position along this length - or beyond it. Then the position of the support will be some addition of the position of the support and some multiple of the pendulum's relative to that support.**

**Label the pendulums 1, 2, and 3. They have support positions $x_{support,i}$, and positions relative to these of $x_{relative,i}$. The position of the attachment between the pendulums and the harmonograph is a sum of these:**

$$x_i = x_{support,i} + \lambda x_{relative,i} \tag{1}$$

**These three points move through space with time, and each point is connected with a string of length $l_i$ to a common point (where the bob of the harmonograph is placed). This creates an intersection of three spheres problem. Fortunately, Wikipedia (/wiki/Trilateration) gives a solution for this:**

**Given three spheres with centers at $(0,0,0)$, $(d,0,0)$ and $(i,j,0)$ and radii $r_1$, $r_2$ and $r_3$ then the position of the lower intersection of the spheres (provided an intersection exists) is:**

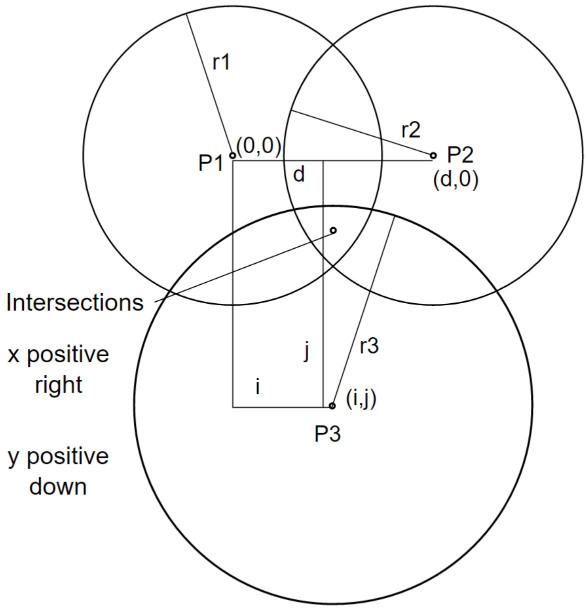



**Diagram and solutions from Wikipedia, thanks.**

$$(a, b, c) \tag{2}$$

where:

$$a = \frac{r_1^2 - r_2^2 + d^2}{2d} \tag{3}$$

$$b = \frac{r_1^2 - r_3^2 + i^2 + j^2}{2j} - \frac{i}{j}a \tag{4}$$

$$c = -\sqrt{r_1^2 - a^2 - b^2} \tag{5}$$

**Therefore, as the three points must be co-planar, and the first vector direction is defined, then the values $d$, $i$ and $j$ need to be found using a transformation, then the results re-transformed to give the position of the bob of the harmonograph in global co-ordinates.**

**The co-ordinates of the three centers of the spheres are at $x_i$, and the lengths of the strings joining the pendulum to the harmonograph bob are the radii of the spheres $r_i$.**

**The first vector of the transformed co-ordinate passes between $x_1$ (the origin) and $x_2$ (d,0,0). Then set:**

$$\underline{a} = x_2 - x_1 \tag{6}$$

**The third vector is perpendicular to the plane containing the points $x_i$, so set as:**

$$\underline{c} = (x_2 - x_1) \times (x_3 - x_1) \tag{7}$$

**The first vector crossed with the second gives the third, therefore, the second vector is:**

$$\underline{b} = (x_2 - x_1) \times ((x_2 - x_1) \times (x_3 - x_1)) \tag{8}$$

**Given these coordinates, the parameters for the intersecting spheres problem given above can be found:**

$$d = (x_2 - x_1) \cdot \hat{\underline{a}} \tag{9}$$

$$i = (x_3 - x_1) \cdot \hat{\underline{a}} \tag{10}$$

$$j = (x_3 - x_1) \cdot \hat{\underline{b}} \tag{11}$$

**N.B. $\hat{\underline{a}}$ is the vector $\underline{a}$ with normalised length.**

**From here the values of $a$, $b$ and $c$ can be found, then the position of the point of the harmonograph found in the global co-ordinates. Then:**

$$x = \underline{x_{support}} + \lambda \underline{x_1} + a\hat{\underline{a}} + b\hat{\underline{b}} + c\hat{\underline{c}} \tag{12}$$

**This harmonograph exists in 3D. To make a more traditional output, the 3D coordinates found here would be projected onto some plane - as occurs (roughly speaking) for a normal harmonograph that draws on paper. However, the use of a 'virtual' harmonograph allows manipulations that might not be feasible with a physical system.**